

6.2 Optimization I

Wednesday, June 12, 2019 7:13 AM

Identify two positive numbers whose sum is 20 and whose product is as large as possible.

- ① Equation A: $x + y = 20$
(uses the given/limiting value)
- ② Equation B: $P = x \cdot y$
(value to max/min)
- ③
$$\begin{array}{r} x + y = 20 \\ -x \quad -x \\ \hline y = 20 - x \end{array}$$

(isolate one of the variables)
- ④
$$\begin{array}{l} P = x \cdot y \\ P = x \cdot (20 - x) \\ \boxed{P = 20x - x^2} \end{array}$$
 ← rewrite Equation B in terms of a single variable
- ⑤
$$P = 20x - x^2 \rightarrow P' = 20 - 2x \rightarrow 20 - 2x = 0 \rightarrow \boxed{x = 10}$$

(derive) (set equal to zero)
- ⑥
$$\boxed{x = 10}$$

$$x + y = 20 \rightarrow 10 + y = 20 \rightarrow \boxed{y = 10}$$

$$P = x \cdot y \rightarrow P = 10 \cdot 10 = \boxed{100}$$

Identify two numbers between $[-20, 5]$ whose product is 225 and whose sum is as large as possible.

- ① $x \cdot y = 225$
- ② $S = x + y$
- ③ $y = \frac{225}{x}$
- ④ $S = x + \frac{225}{x}$
- ⑤
$$S = x + 225x^{-1} \rightarrow S' = 1 - 225x^{-2}$$

$$0 = 1 - \frac{225}{x^2}$$

$$\frac{225}{x^2} = 1 \cdot x^2$$

$$\pm \sqrt{225} = \sqrt{x^2}$$

$$\pm 15 = x$$
- ⑥

x	y	S
15		
-15	-15	-30
-20	-11.25	-31.25
5	45	

✓ -15, -11.25

$$\begin{array}{l} \text{ } \\ \text{ } \end{array} \left\{ \begin{array}{l} x = -15 \\ y = -15 \\ \text{sum} = -30 \end{array} \right. \quad \pm 15 = x$$